

# Bidirectional Path Tracing Using Backward Stochastic Light Culling

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Stochastic light culling for glossy caustics  
[Tokuyoshi17]

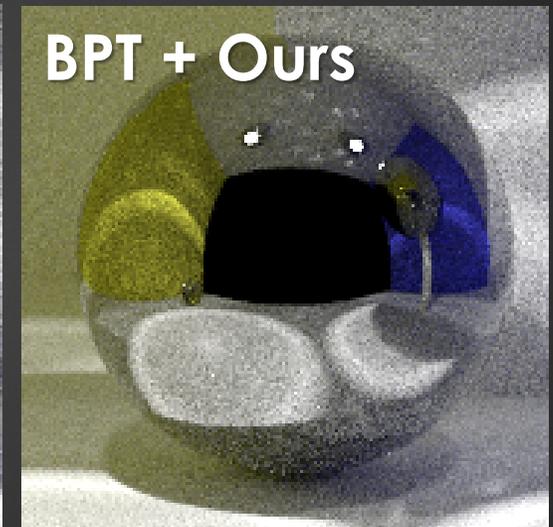
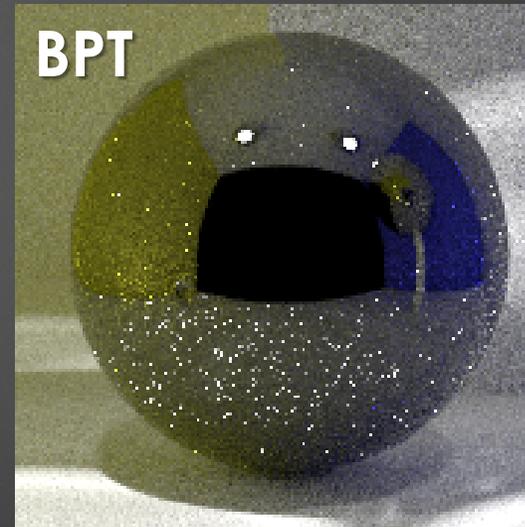
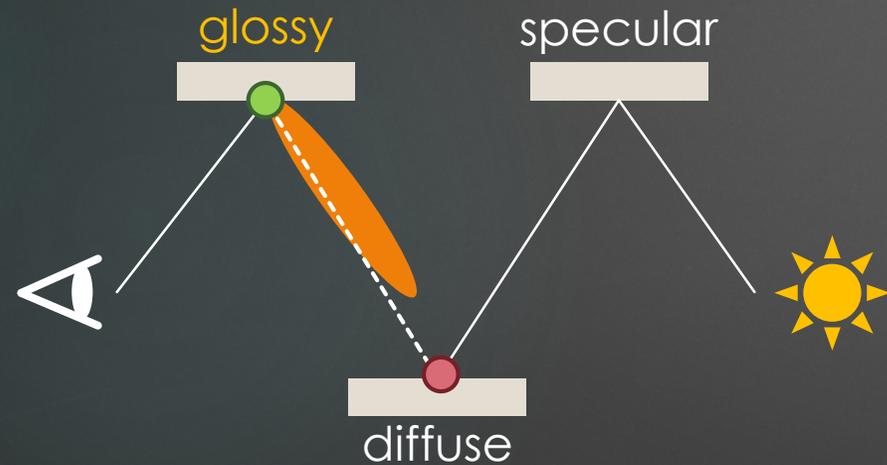
Real-time rendering



Offline bidirectional path tracing  
(BPT)

# Specular-Diffuse-Glossy Paths in BPT

- ▶ Connectable
- ▶ Inefficient for **very highly glossy** reflections ☹️

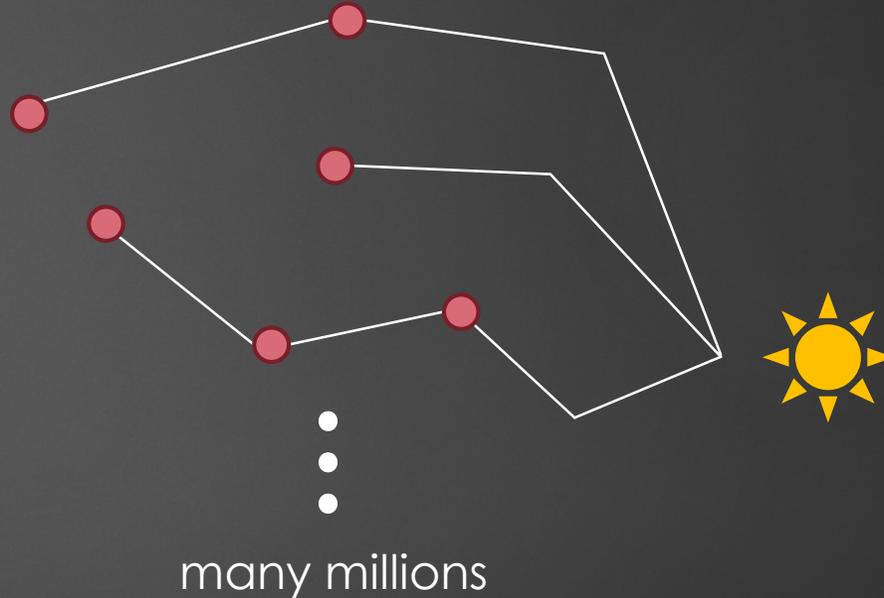


GGX roughness: 0.001

# Algorithm Overview



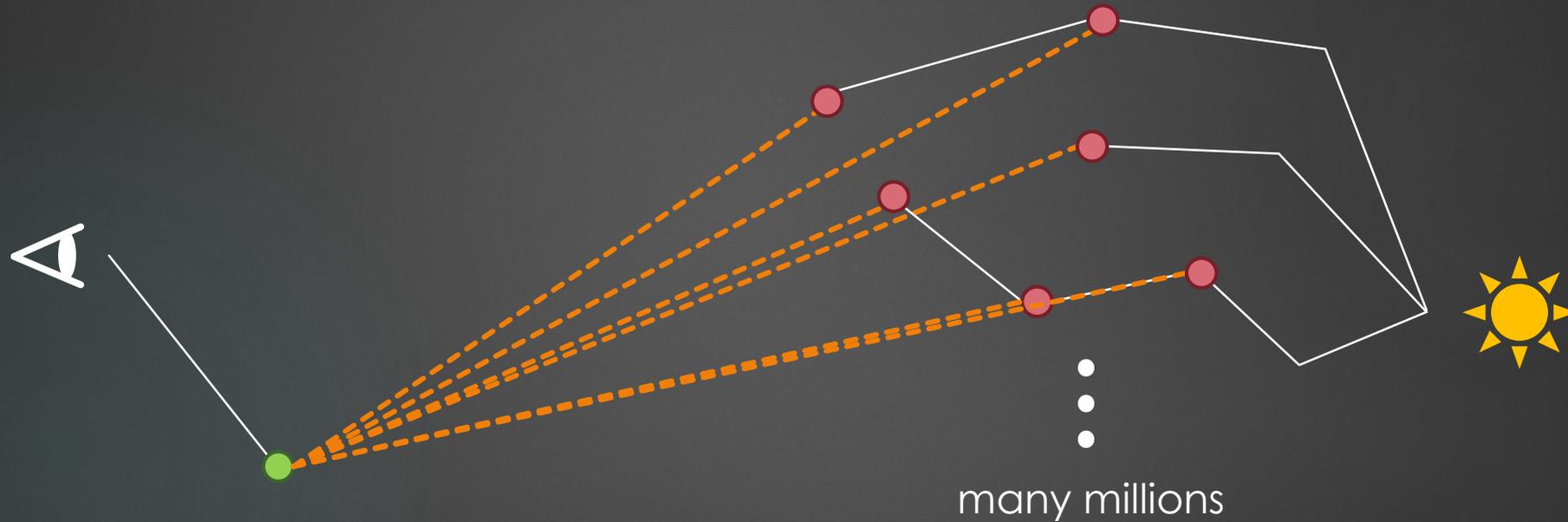
Light-subpath tracing pass



Store many light vertices in a cache similar to virtual point lights [Keller97]

# Algorithm Overview

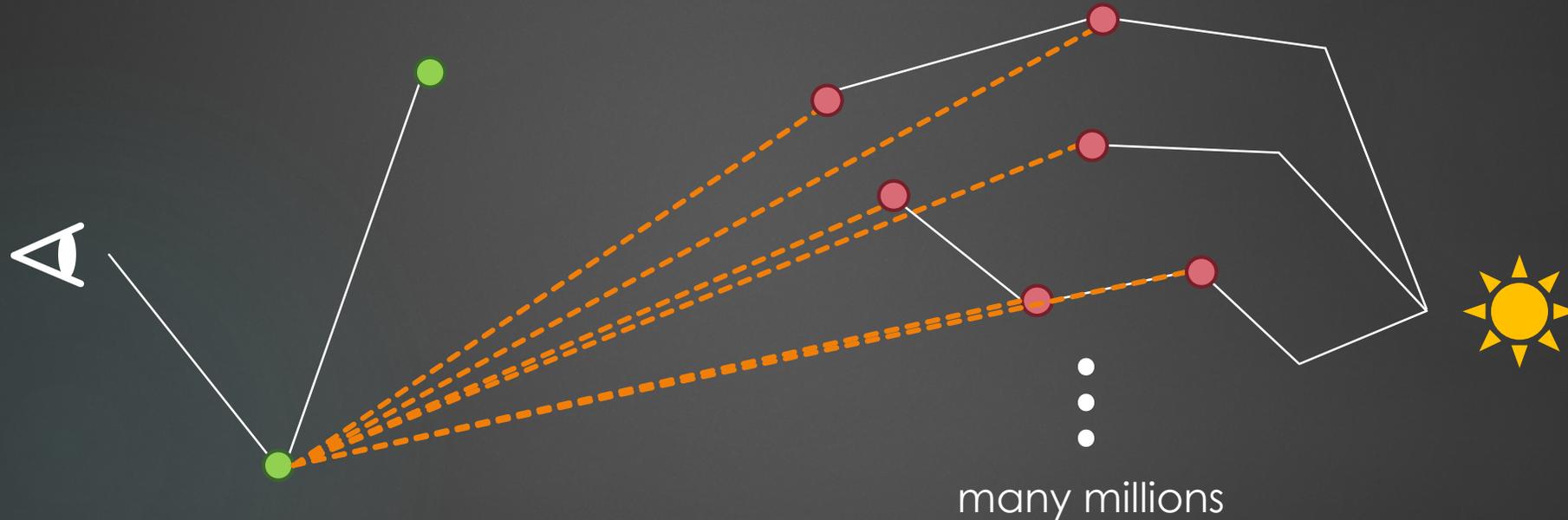
Eye-subpath tracing pass



Russian roulette [Arvo90] for all the cached light vertices

# Algorithm Overview

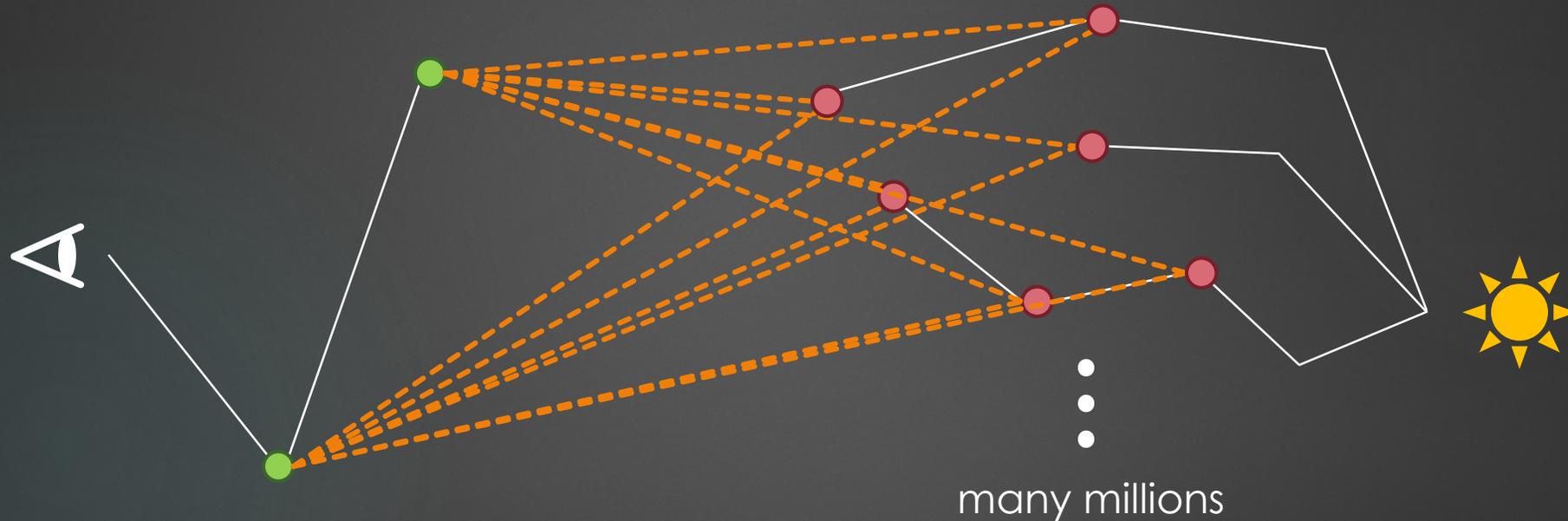
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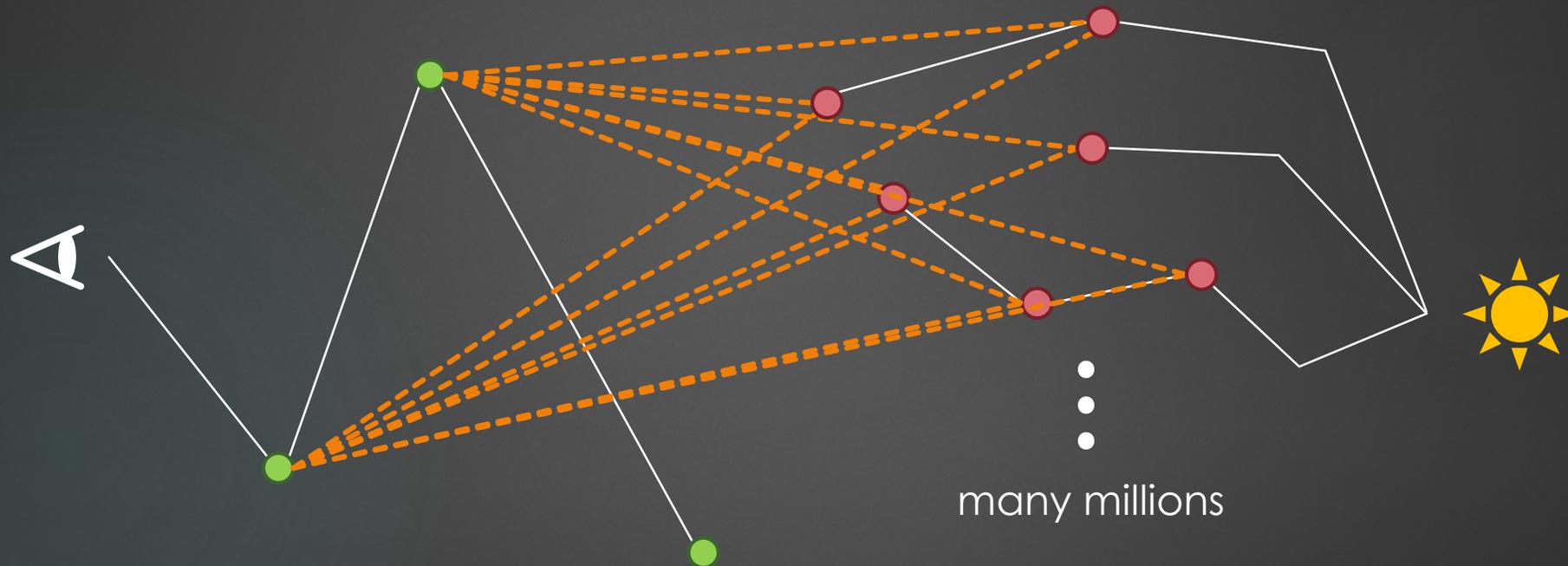
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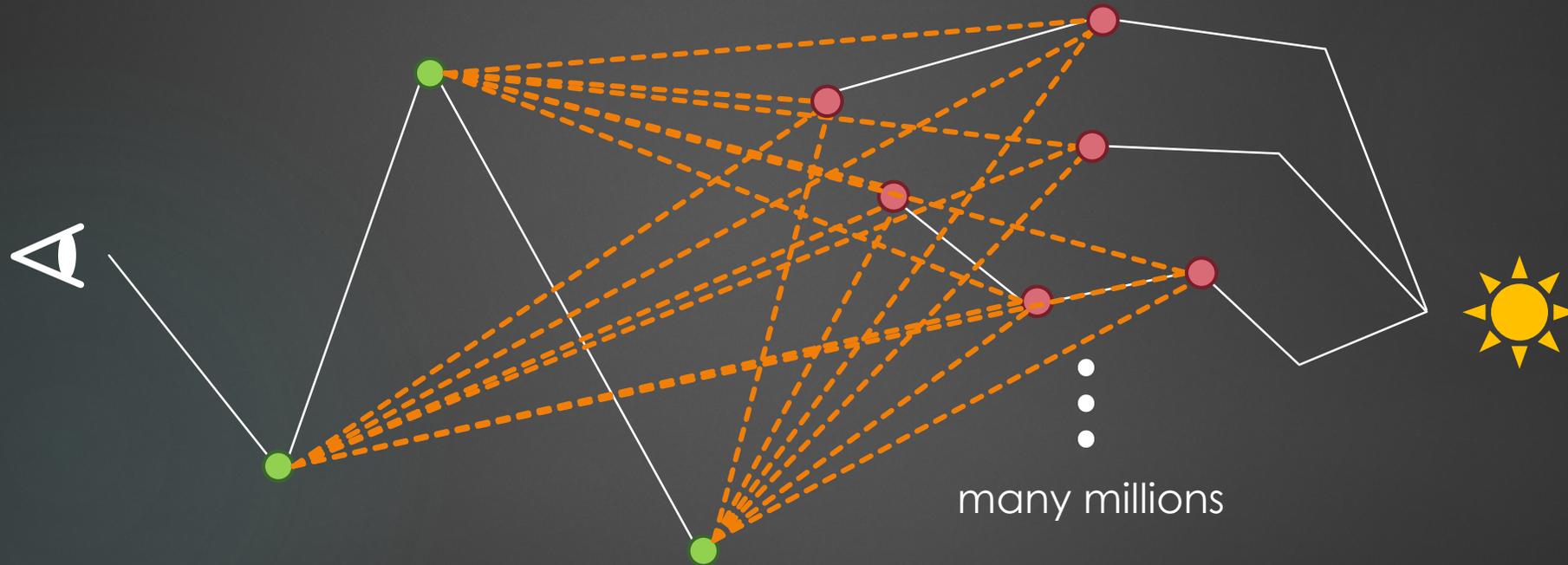
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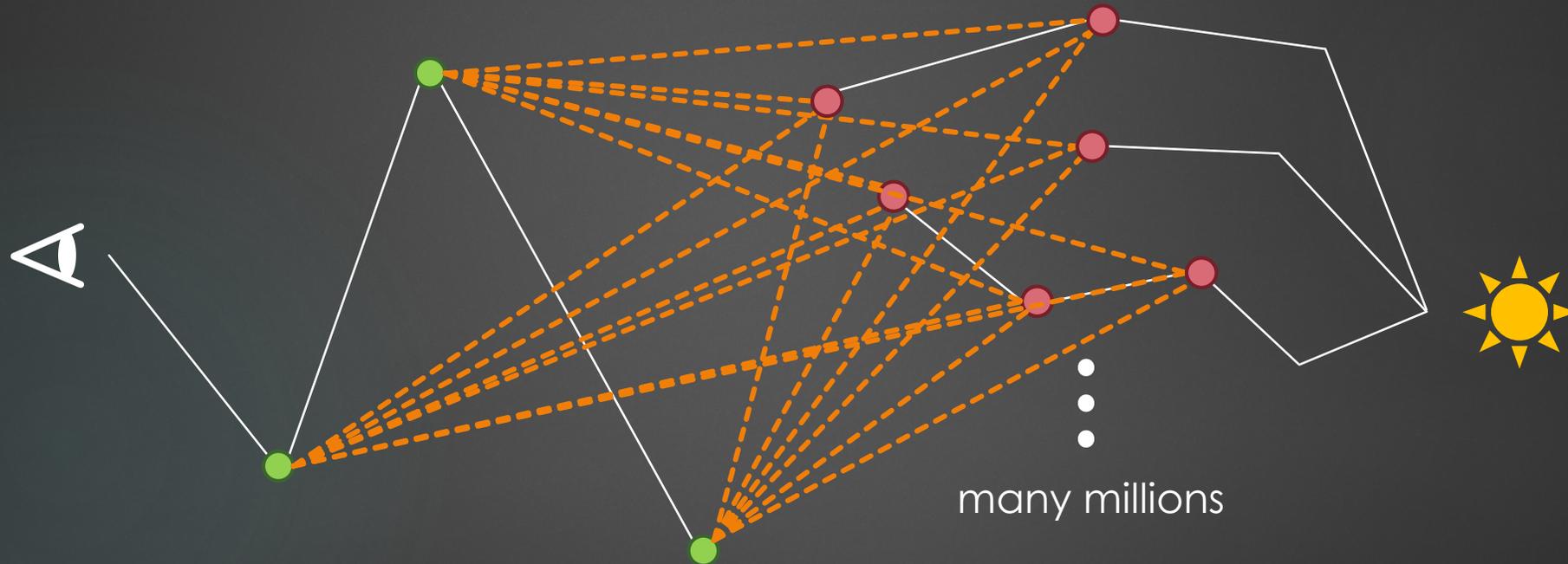
Eye-subpath tracing pass



Russian roulette [Arvo90] for all the cached light vertices

# Algorithm Overview

Eye-subpath tracing pass

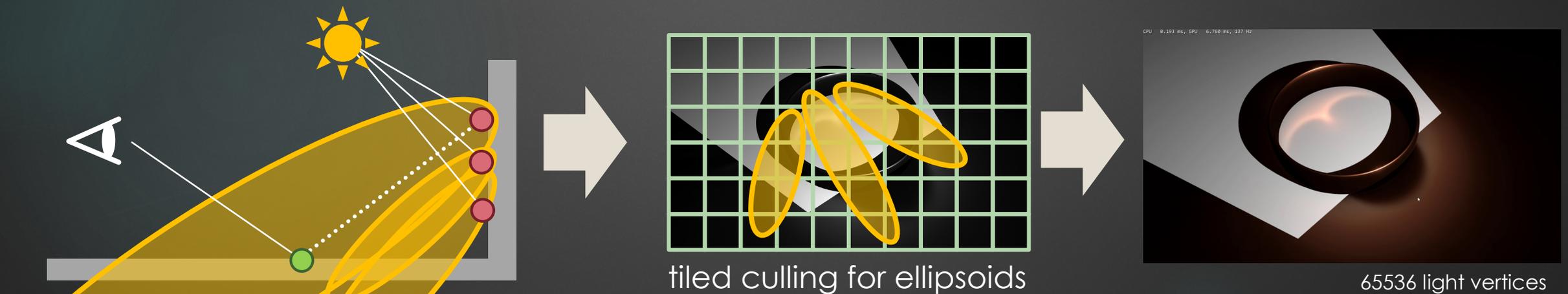


Russian roulette [Arvo90] for all the cached light vertices

**Accelerated by stochastic light culling**

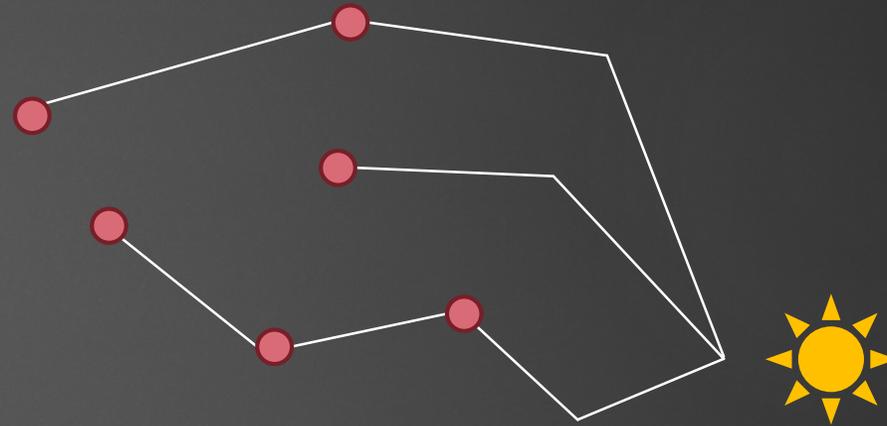
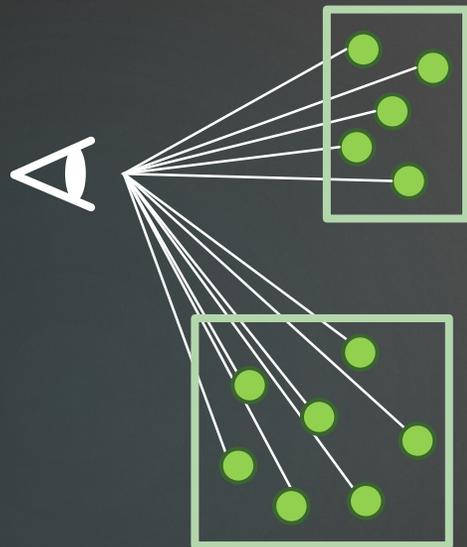
# Stochastic Light Culling [Tokuyoshi16]

- ▶ Restrict the range of influence of a light vertex based on **Russian roulette**
- ▶ Ellipsoidal range for glossy reflections
  - ▶ Computable analytically for Phong and GGX BRDFs [Dachsbacher06, Tokuyoshi17]
- ▶ Real-time culling algorithms are available (e.g., tiled culling)



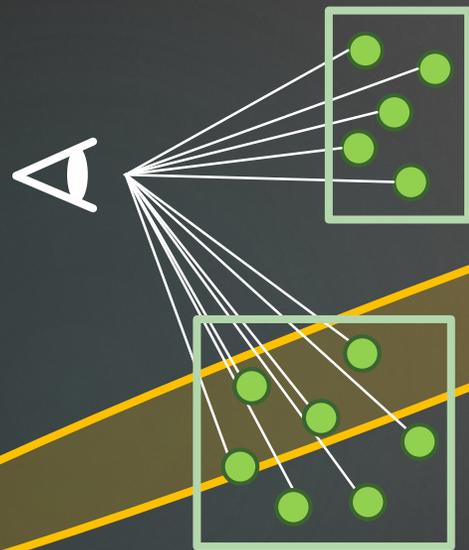
# Tiled Culling

Bounding volume for eye vertices grouped in screen-space

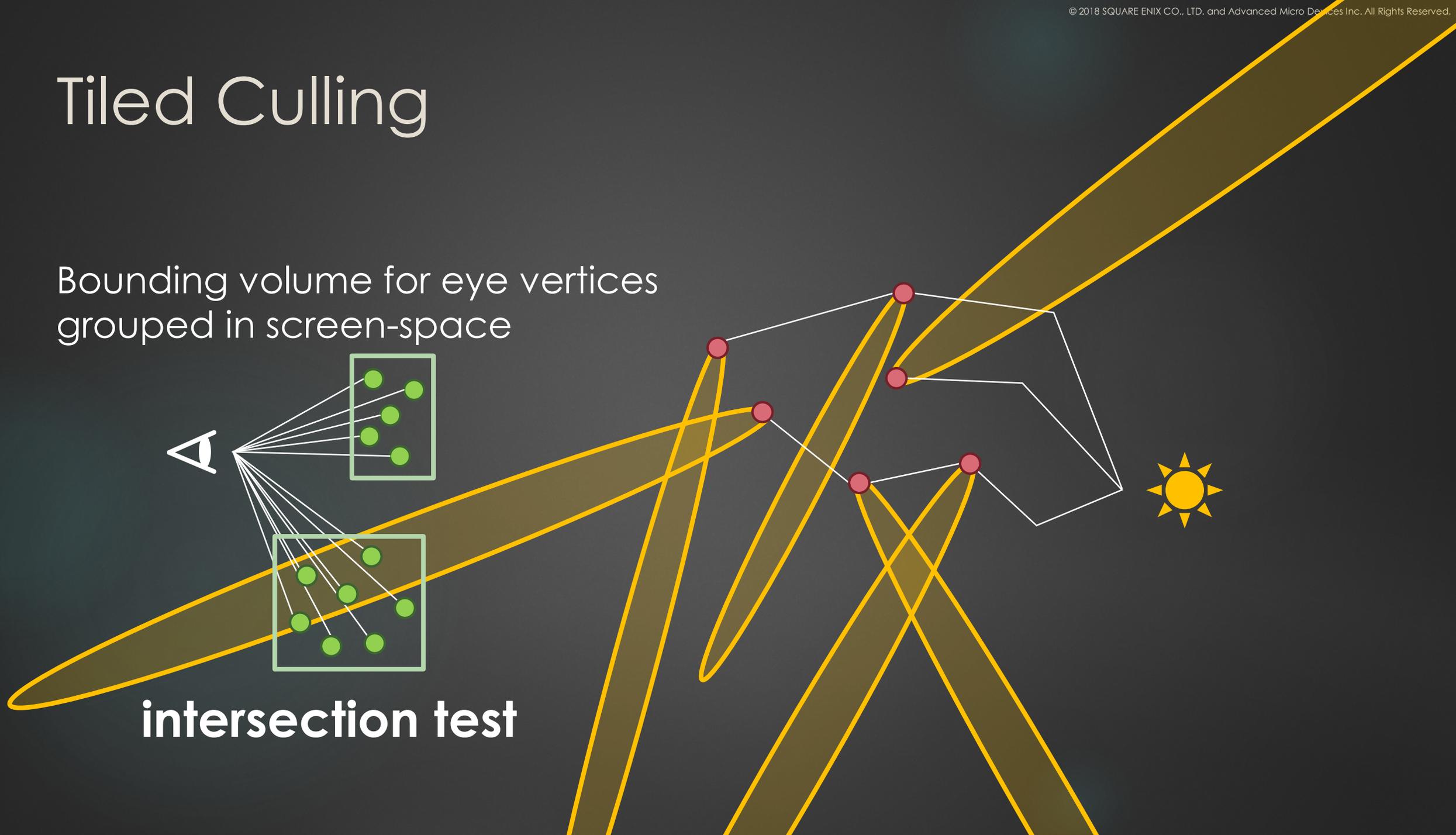


# Tiled Culling

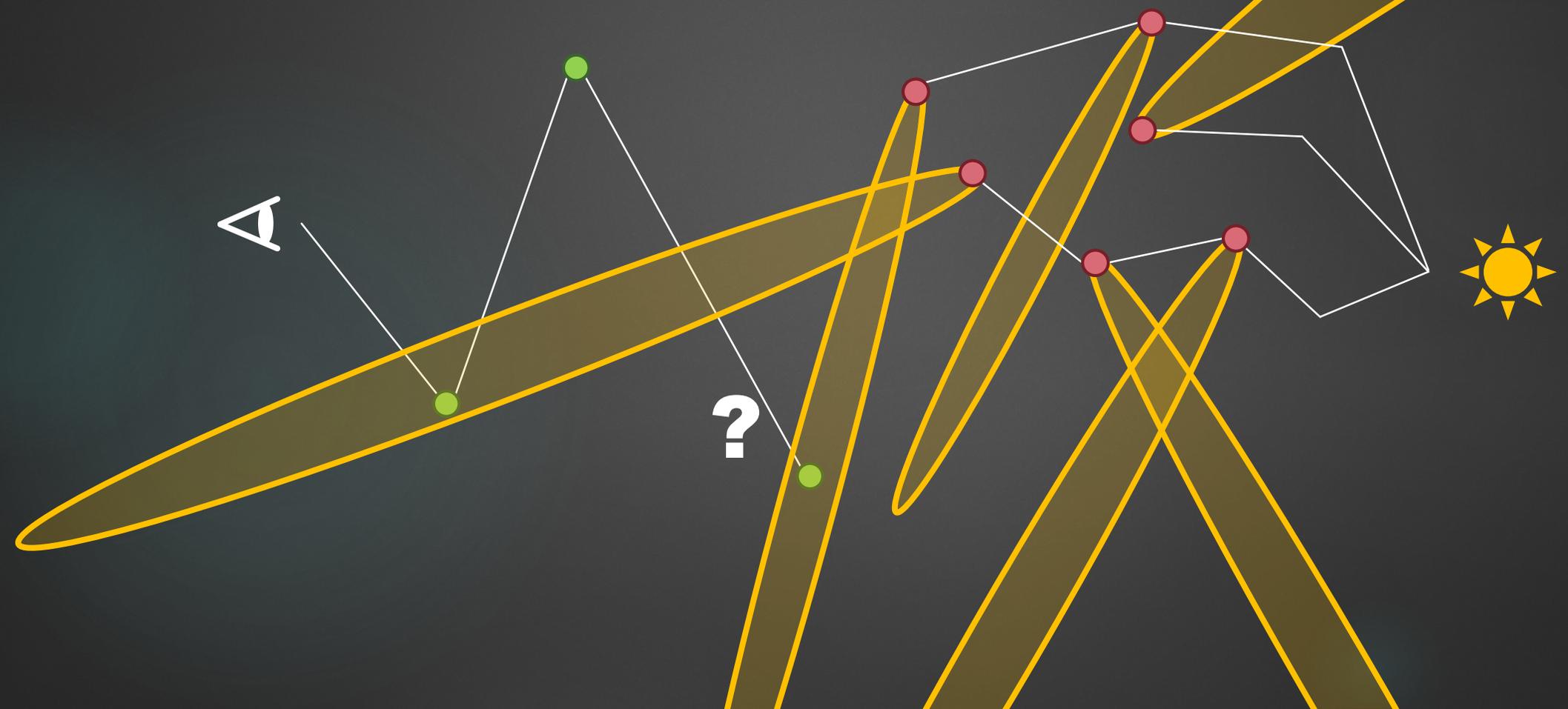
Bounding volume for eye vertices grouped in screen-space



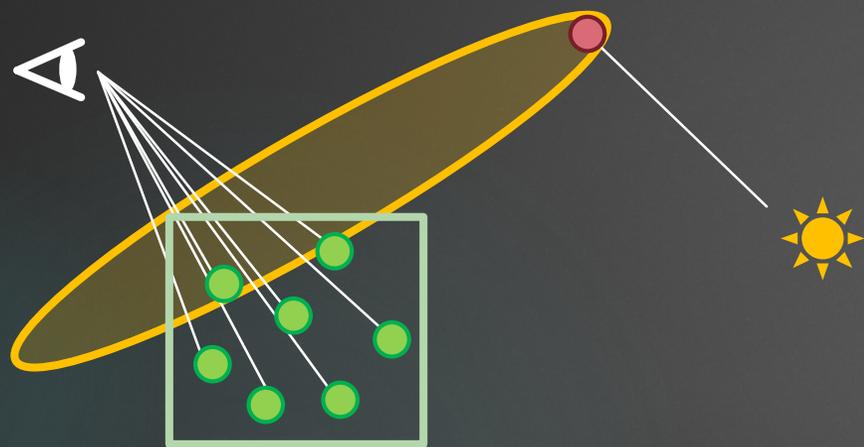
**intersection test**



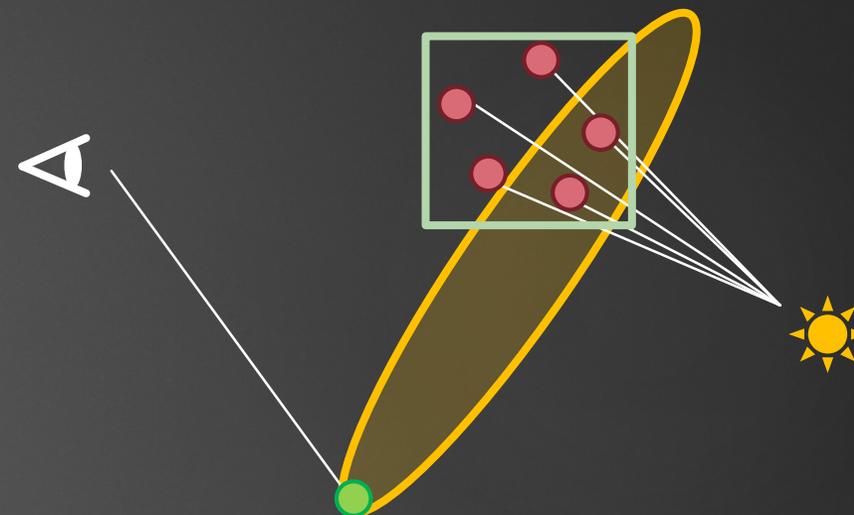
# Problem: Indirect Eye Vertices



# Opposite Approach



Range from each **light** vertex  
&  
Bounding volume for **eye** vertices



Range from each **eye** vertex  
&  
Bounding volume for **light** vertices

# Range from an Eye Vertex

- ▶ Probability of Russian roulette:

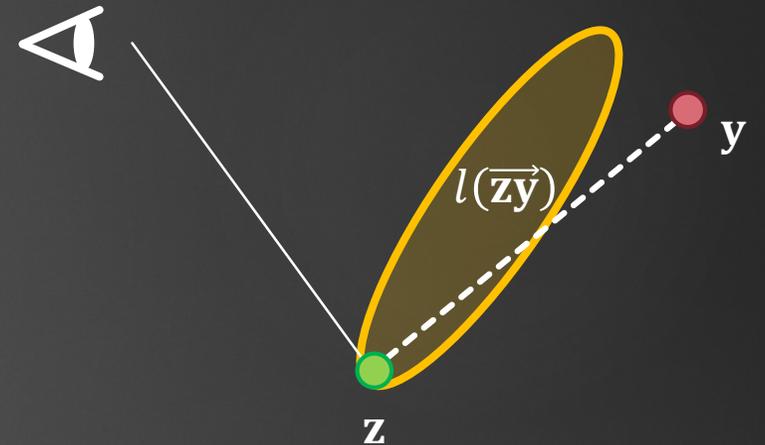
user-defined parameter      GGX lobe

$$P(\mathbf{z}, \mathbf{y}) = \min \left( \frac{Cq(\overline{\mathbf{z}\mathbf{y}})}{\|\mathbf{y} - \mathbf{z}\|^2}, 1 \right)$$

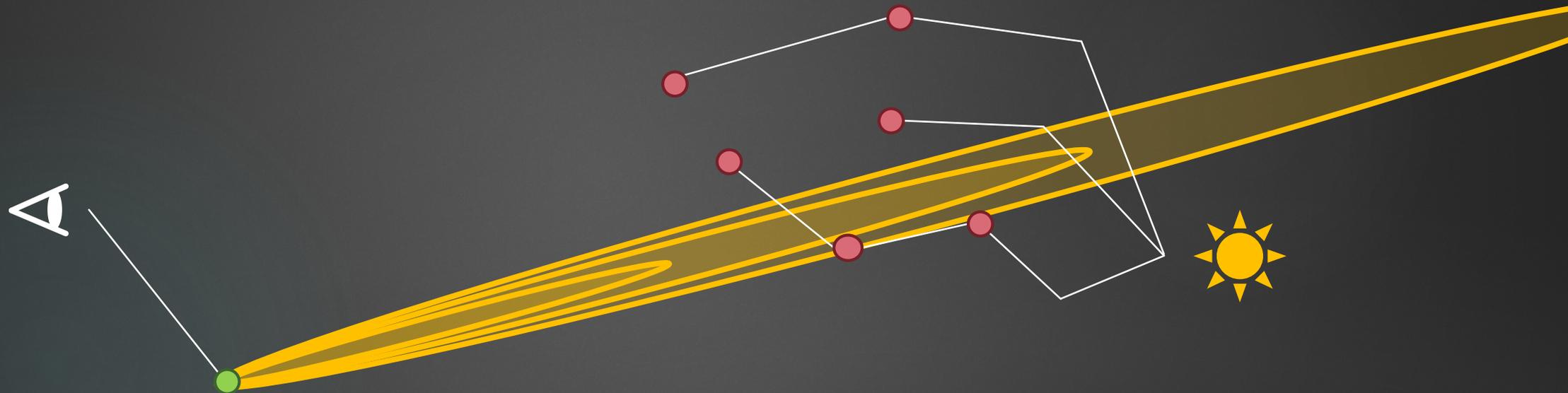
- ▶ Acceptance range from the eye vertex:

$$l(\overline{\mathbf{z}\mathbf{y}}) \geq P^{-1}(\xi) = \sqrt{\frac{Cq(\overline{\mathbf{z}\mathbf{y}})}{\xi}}$$

uniform random number  $\in [0,1)$  generated for each light vertex

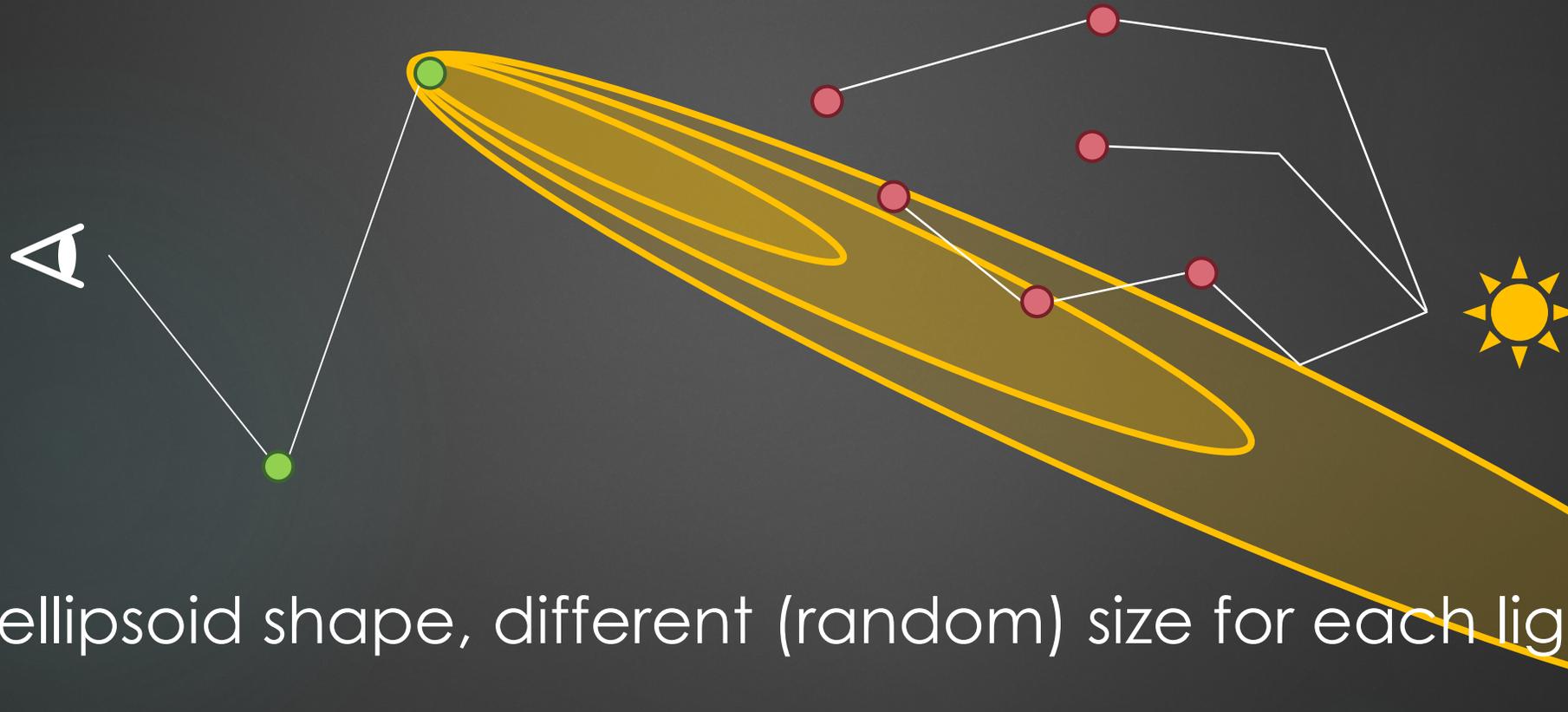


# Backward Stochastic Light Culling



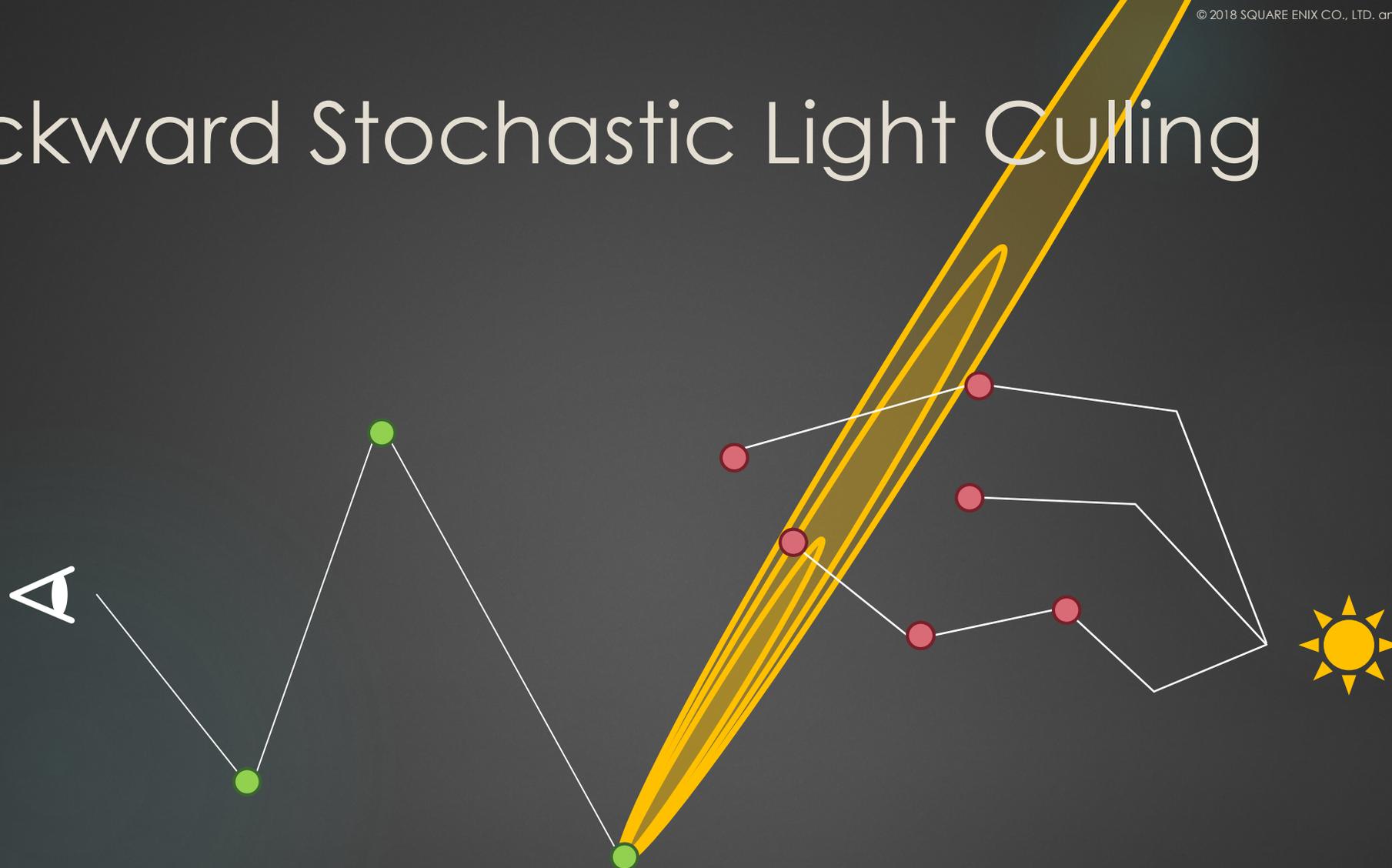
Same ellipsoid shape, different (random) size for each light vertex

# Backward Stochastic Light Culling



Same ellipsoid shape, different (random) size for each light vertex

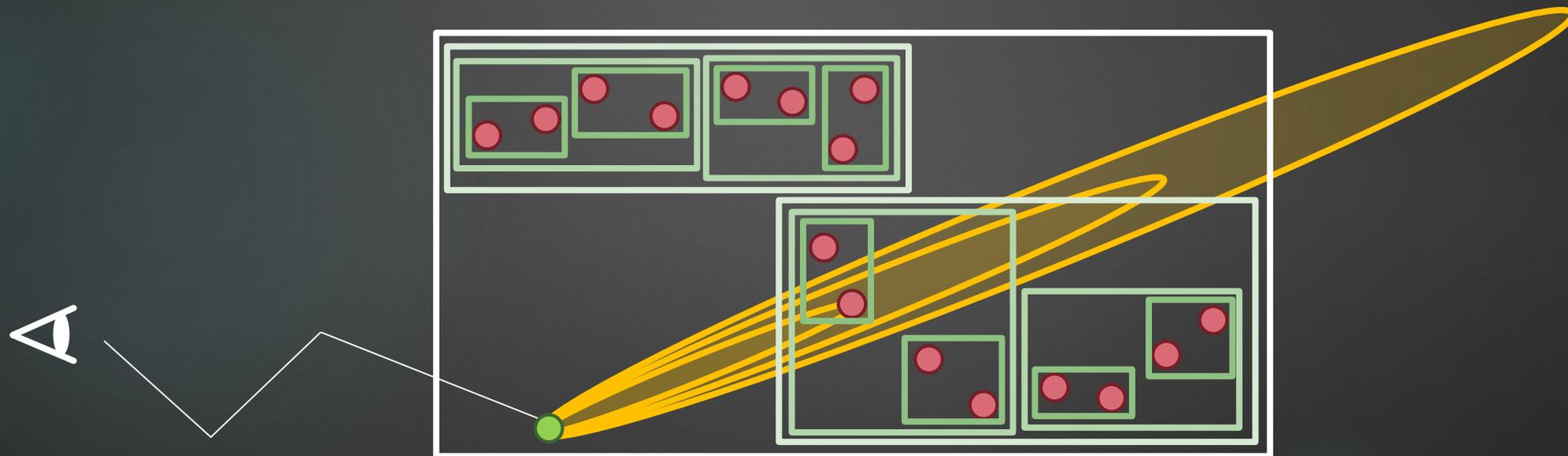
# Backward Stochastic Light Culling



Same ellipsoid shape, different (random) size for each light vertex

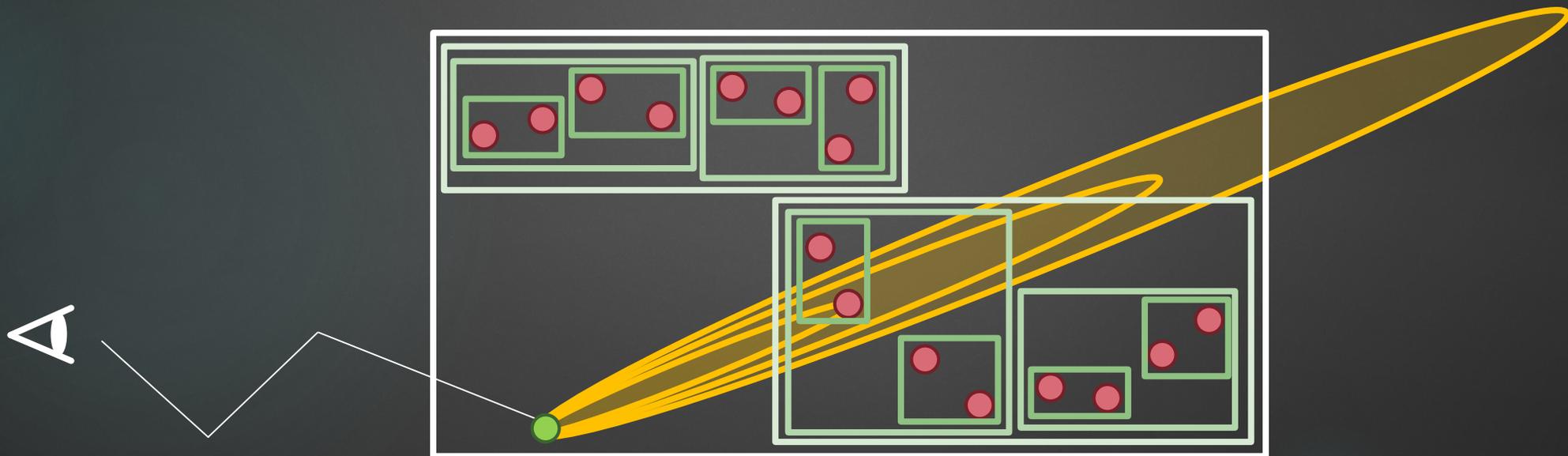
# Culling Using BVH

- ▶ Build BVH for cached light vertices
- ▶ Hierarchical intersection tests between the ellipsoid and each BVH node



# Random Ellipsoid Size

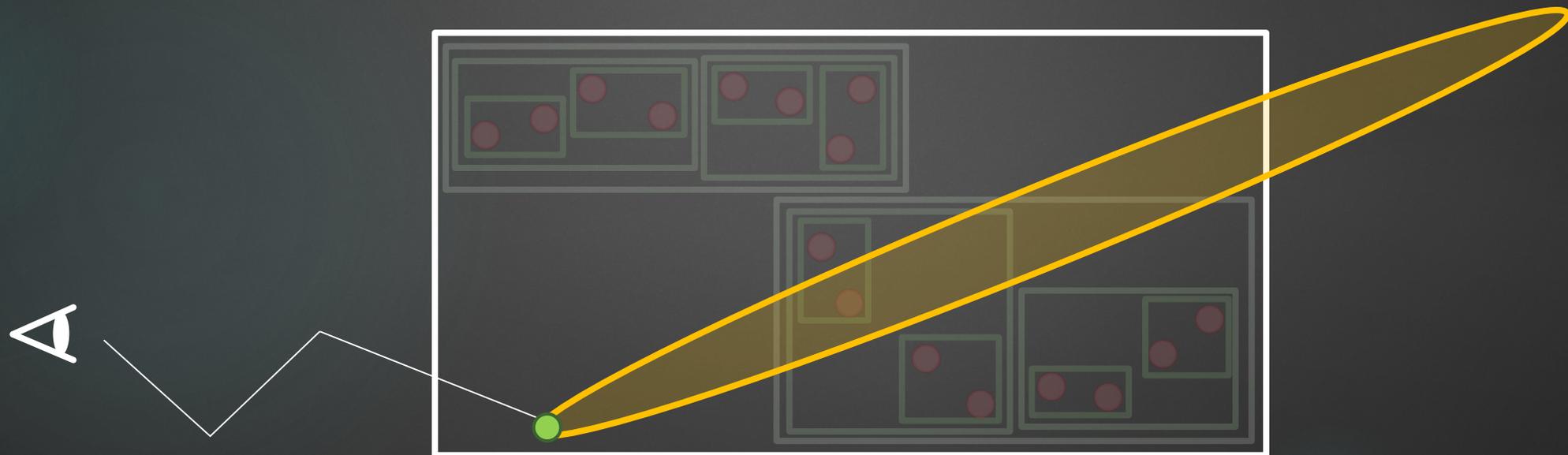
- ▶ Random ellipsoid size for each light vertex (i.e., leaf node)
- ▶ Use the **largest size** in each node for conservative intersection test



# Random Ellipsoid Size

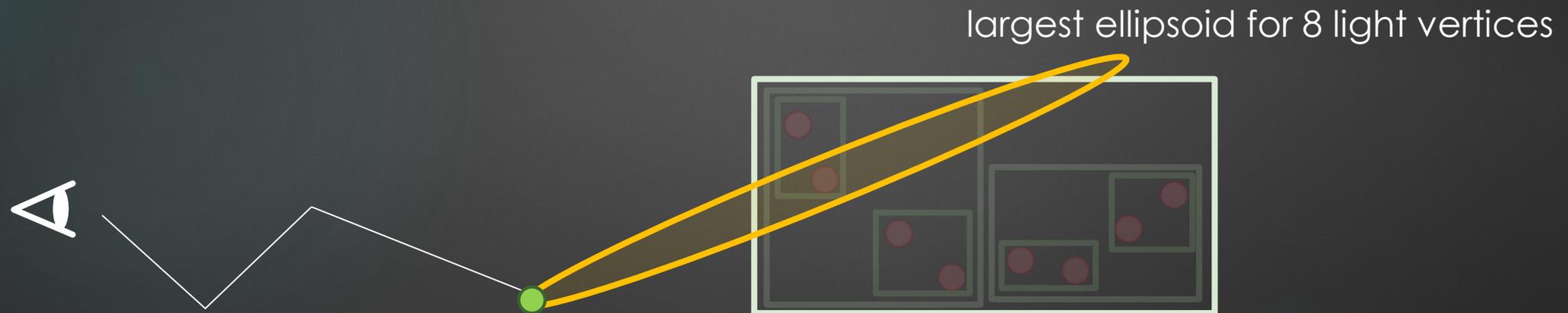
- ▶ Random ellipsoid size for each light vertex (i.e., leaf node)
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largest ellipsoid for 16 light vertices



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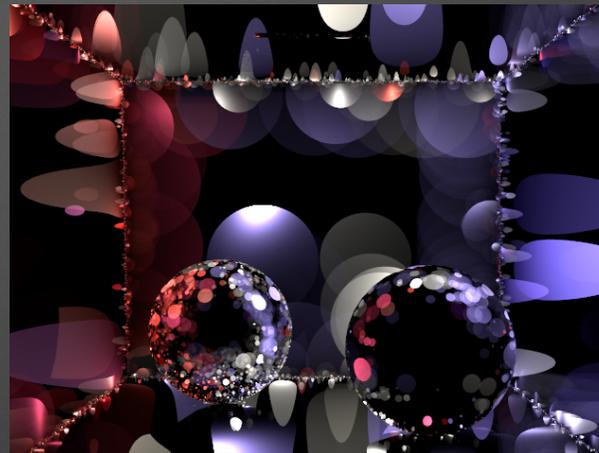
$$\max_{i \in L} l(\omega) = \sqrt{\frac{Cq(\omega)}{\min_{i \in L} \xi_i}}$$

The largest size is given by the **minimum of uniform random numbers**

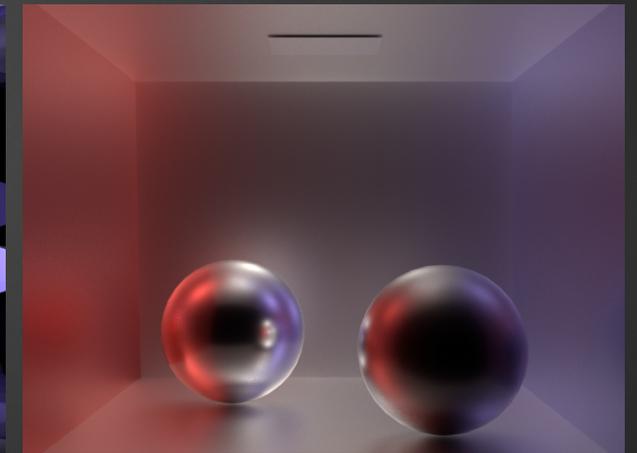
# Precomputing Random Numbers?

- ▶ Assign a **single random number** to each light vertex in preprocessing
  - ▶ Similar to previous stochastic light culling [Tokuyoshi16,17]
- ▶ Store the minimum random number into each BVH node

**Correlation of variance** ☹️



Precomputed



Reference



Generate random numbers **on-the-fly**

# Distribution of the Minimum Random Number

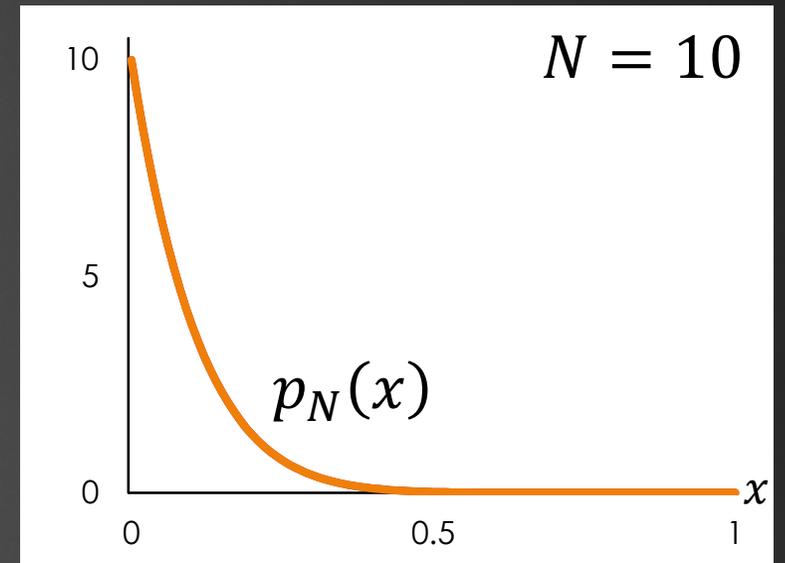
PDF of the minimum value among  $N$  uniform random numbers:

$$p_N(x) = N(1 - x)^{N-1}$$



Inverse of the CDF

$$\min_{i \in [0, N)} \xi_i = c_N^{-1}(\xi) = 1 - (1 - \xi)^{\frac{1}{N}}$$



**Computable using only a single uniform random number  $\xi \in [0,1)$**

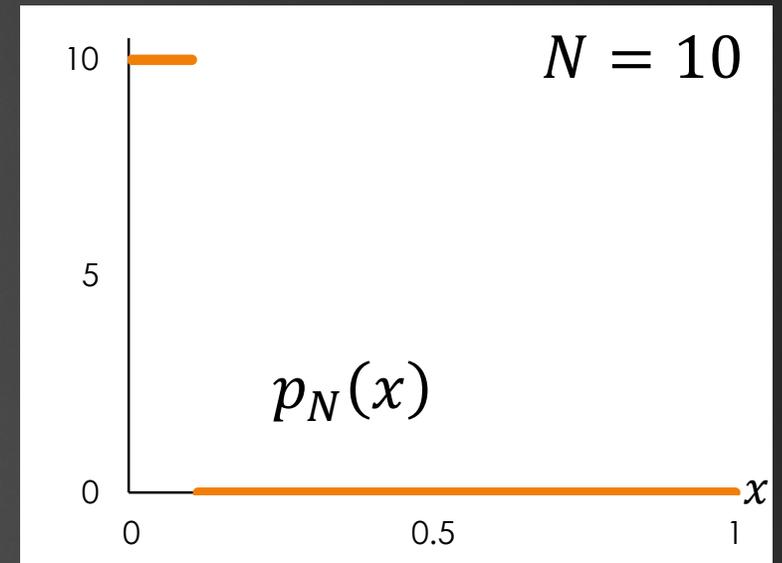
# Case of 1D Stratified Sampling

- ▶ The minimum value is always within the lowest stratum

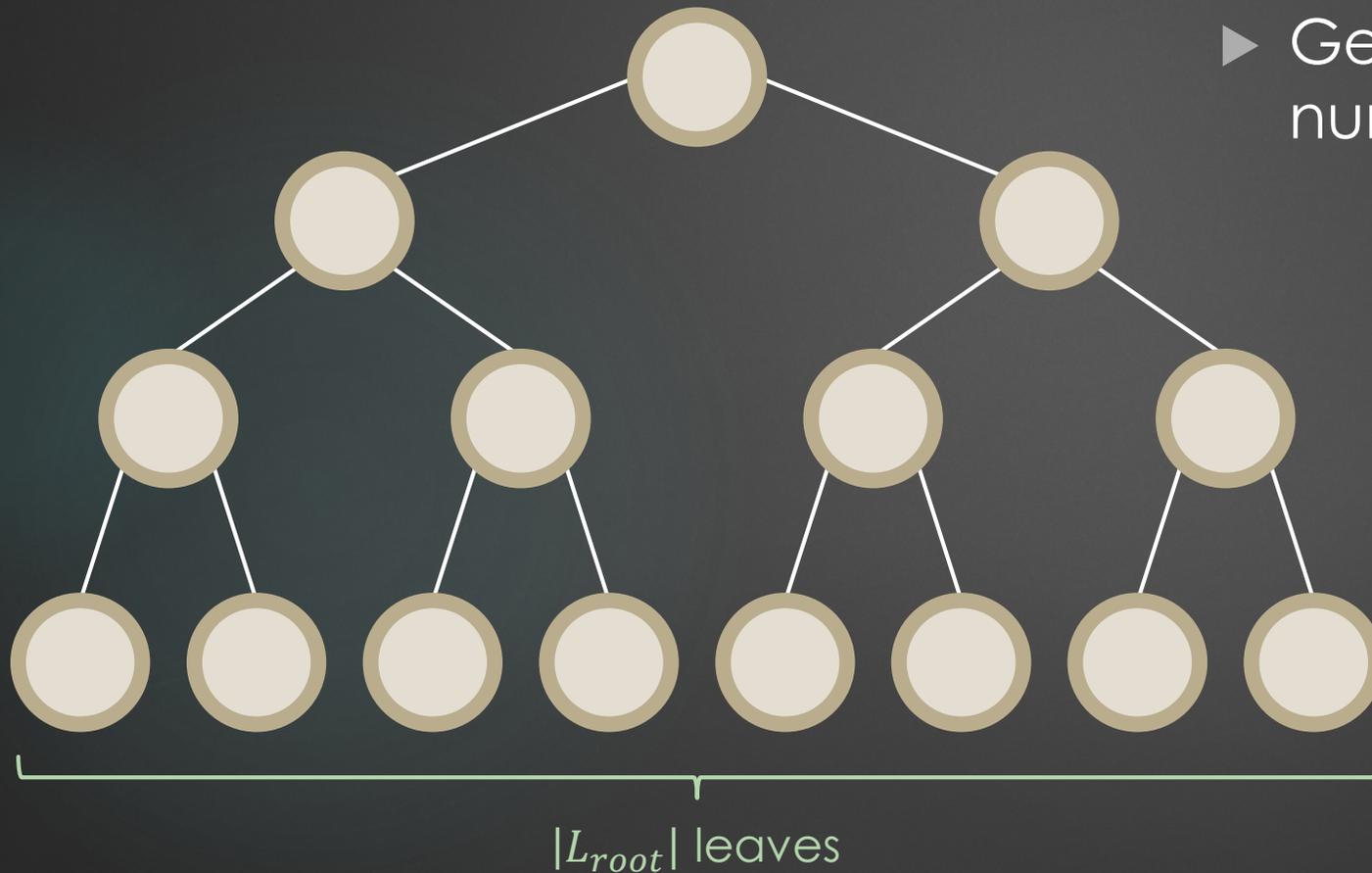


$$\min_{i \in [0, N)} \xi_i = c_N^{-1}(\xi) = \frac{\xi}{N}$$

Simple 😊



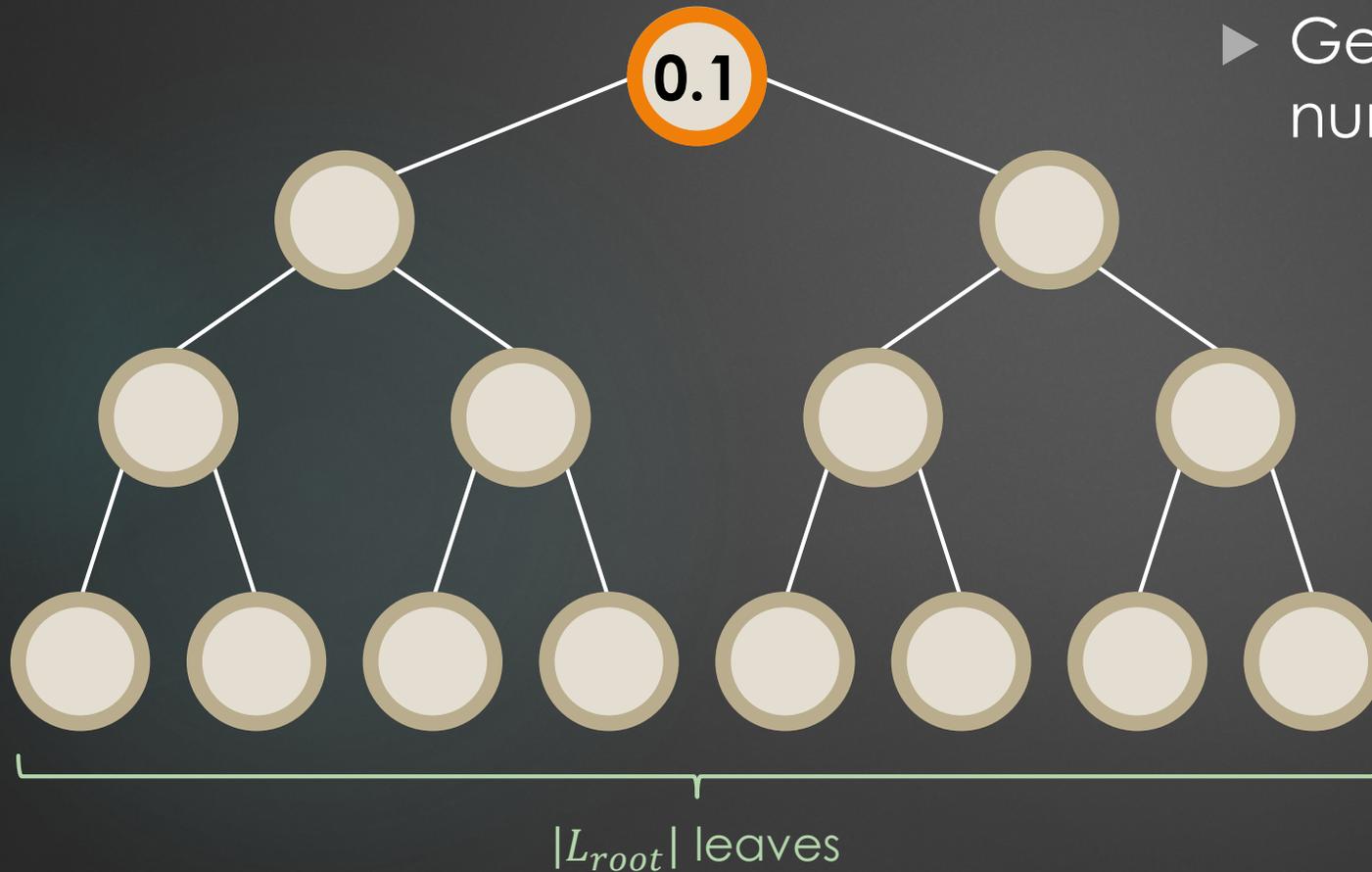
# Semi-Stratified Random Number Assignment in Top-Down BVH Traversal



- Generate a minimum random number at the root node

$$\min_{i \in L_{root}} \xi_i = \frac{\xi}{|L_{root}|}$$

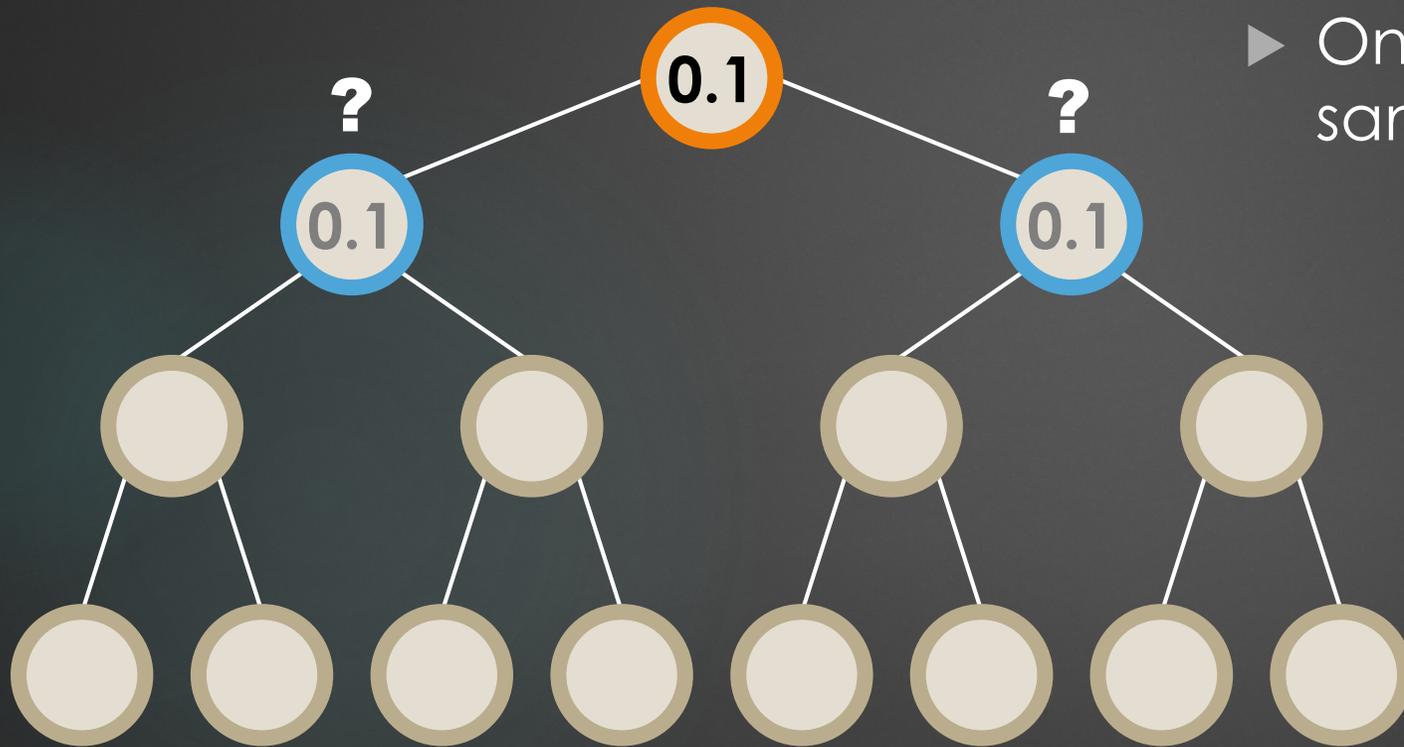
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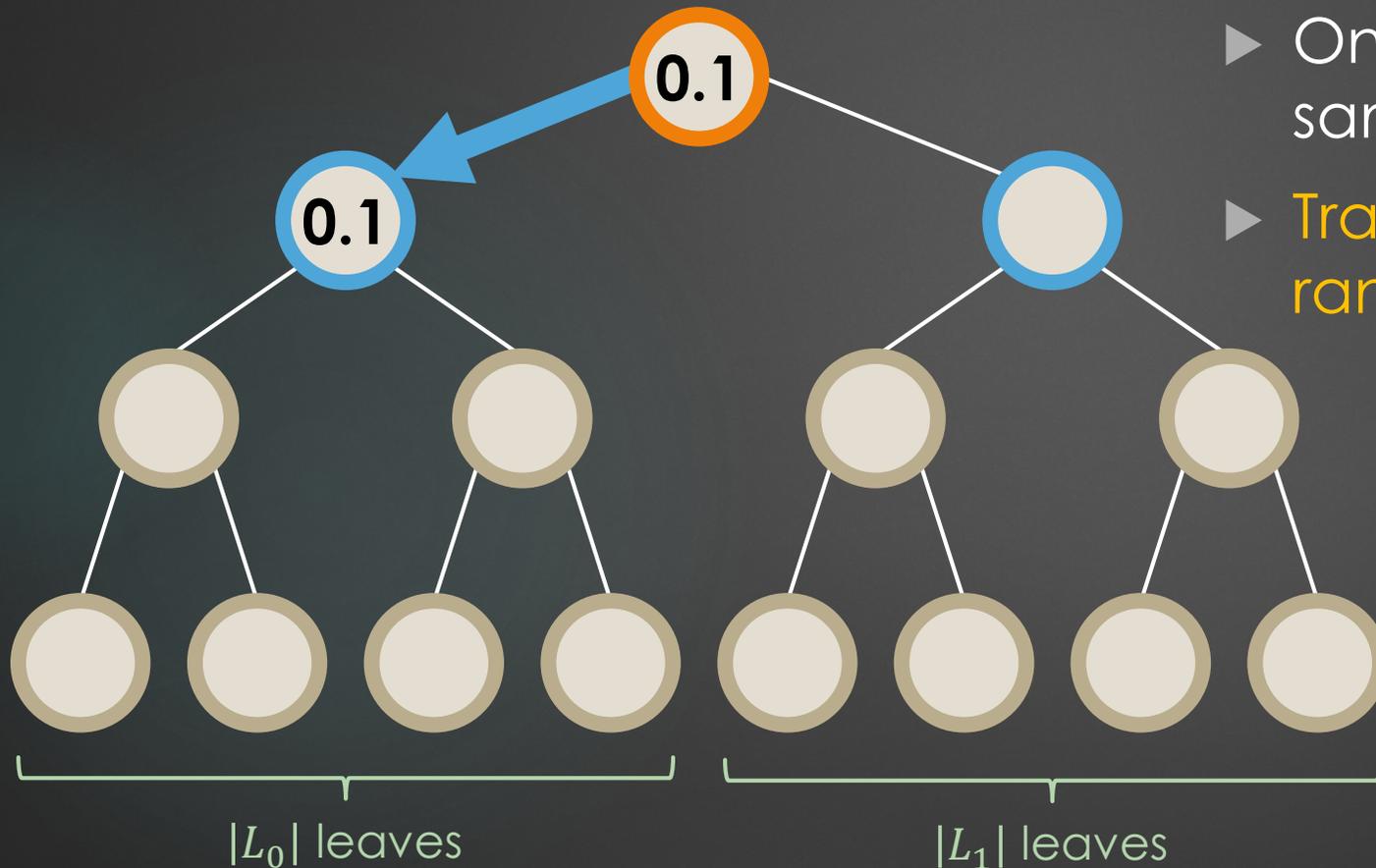
$$\min_{i \in L_{root}} \xi_i = \frac{\xi}{|L_{root}|}$$

# Semi-Stratified Random Number Assignment in Top-Down BVH Traversal



► One of the child values must be the same as the parent's value

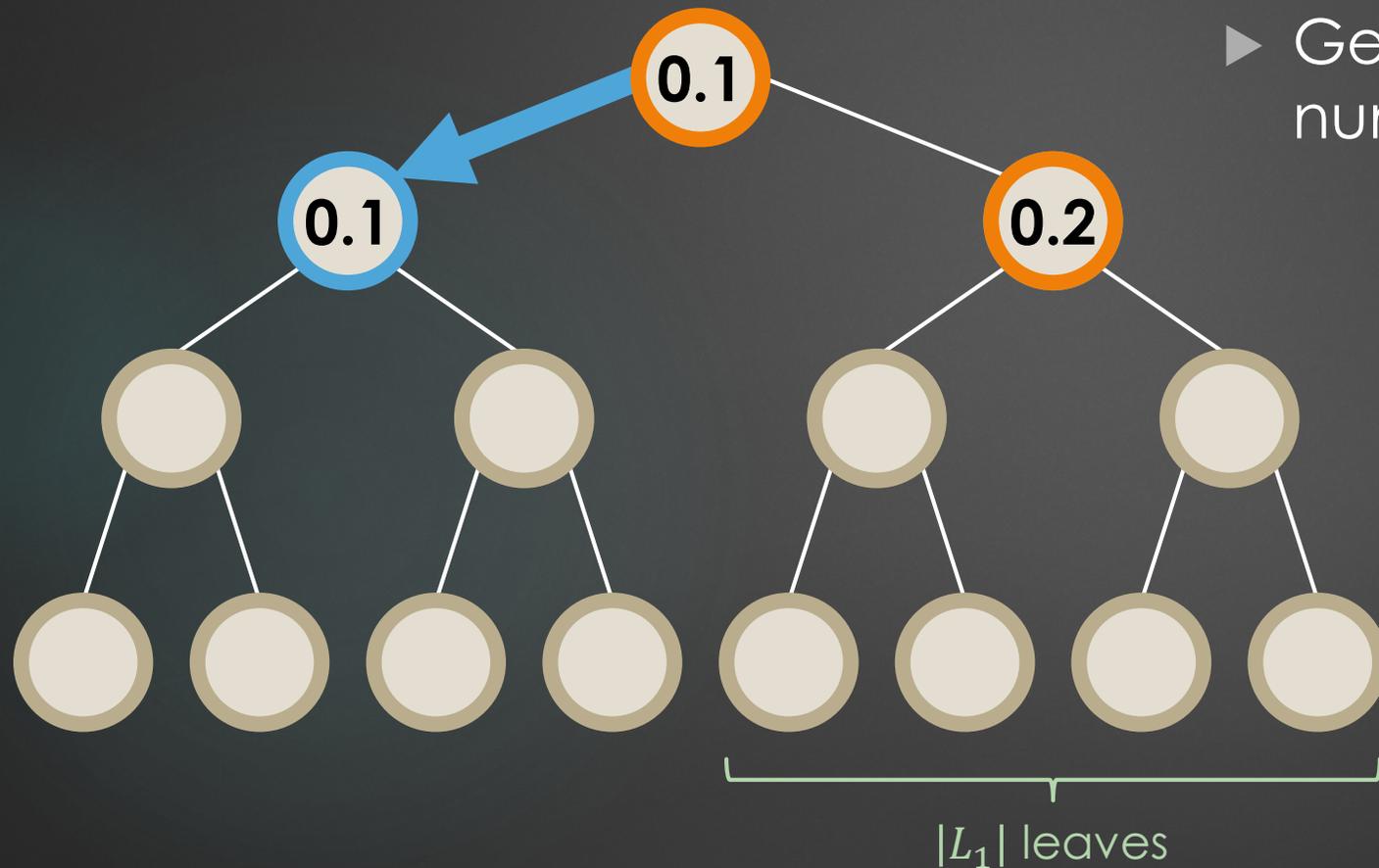
# Semi-Stratified Random Number Assignment in Top-Down BVH Traversal



- ▶ One of the child values must be the same as the parent's value
- ▶ Transmit the parent's value to a randomly selected single child

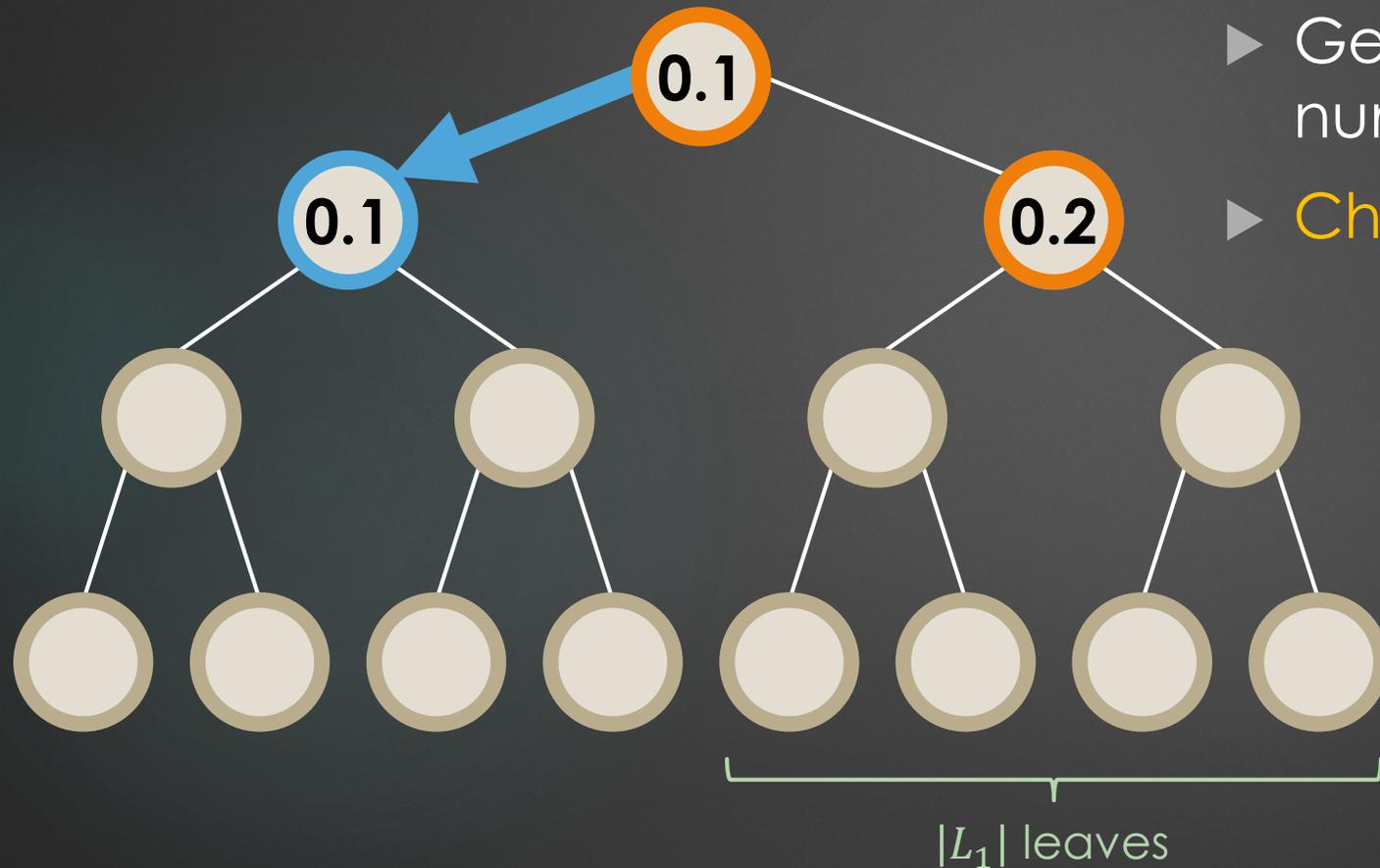
$$\text{Probability} = \frac{|L_0|}{|L_0| + |L_1|}$$

# Semi-Stratified Random Number Assignment in Top-Down BVH Traversal



- Generate a new minimum random number for the other child

# Semi-Stratified Random Number Assignment in Top-Down BVH Traversal



- ▶ Generate a new minimum random number for the other child
- ▶ Child's value  $\geq$  parent's value

$$\min_{i \in L_1} \xi_i = s + (1 - s) \frac{\xi}{|L_1|}$$

Upper bound of the parent's stratum

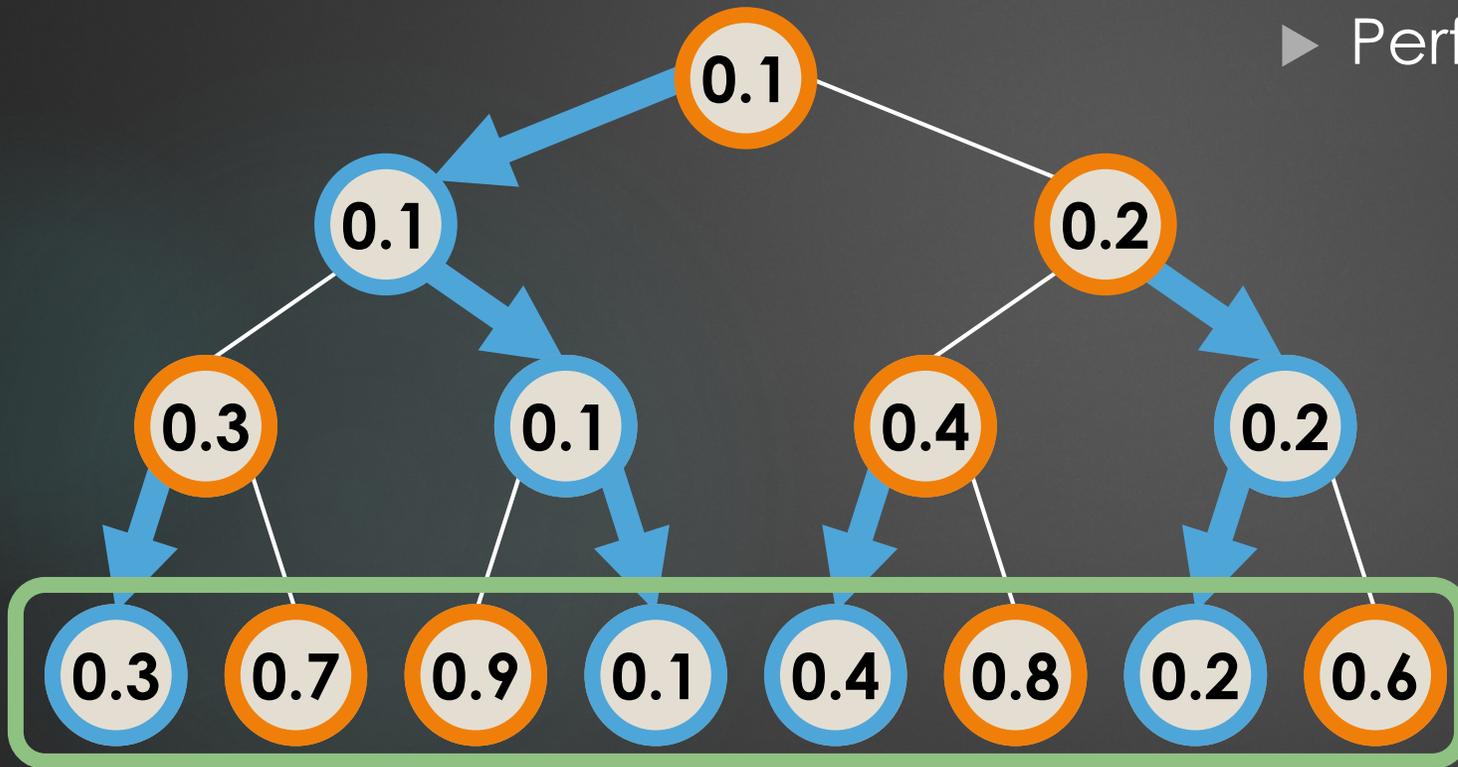
# Semi-Stratified Random Number Assignment in Top-Down BVH Traversal



► Perform recursively in BVH traversal

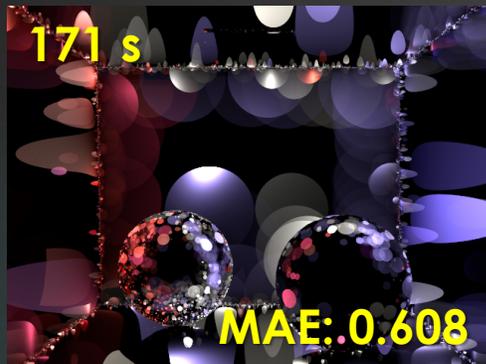
# Semi-Stratified Random Number Assignment in Top-Down BVH Traversal

► Perform recursively in BVH traversal

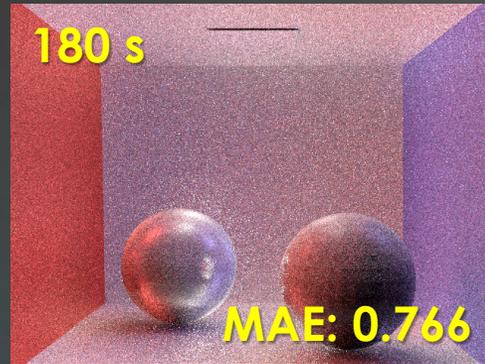


uniform & partially stratified for leaf nodes

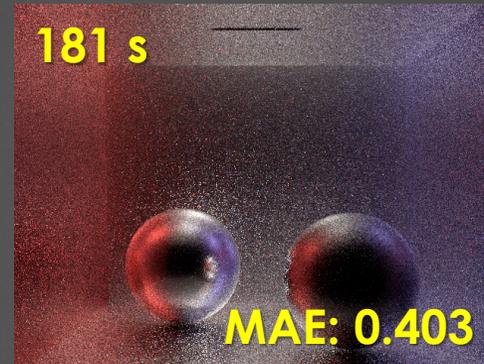
# Comparison of Random Number Assignment Methods



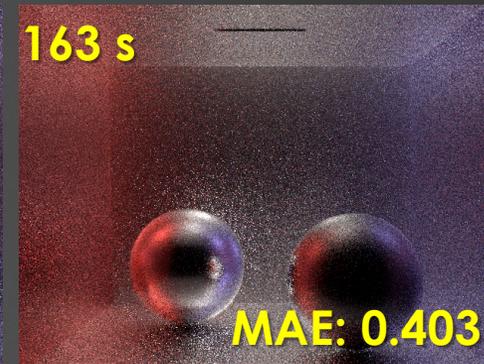
Precomputed



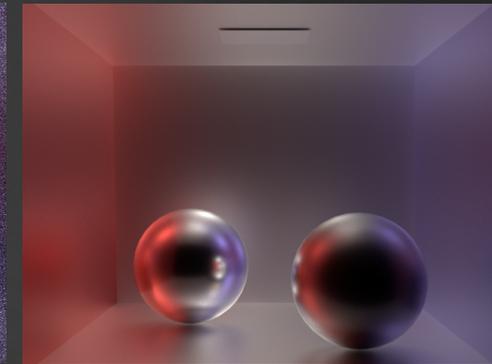
Non-stratified  
(single precision)



Non-stratified  
(double precision)



Semi-stratified  
(single precision)



Reference

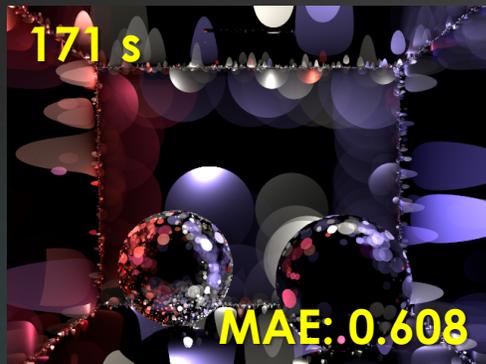
$$c_N^{-1}(\xi) = 1 - (1 - \xi)^{\frac{1}{N}}$$

Noticeable precision error for large  $N$  😞

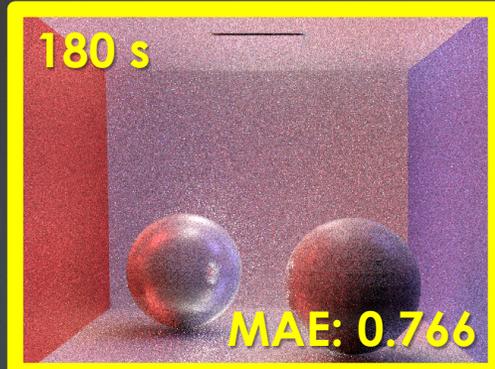
$$c_N^{-1}(\xi) = \frac{\xi}{N}$$

Fast & precise 😊

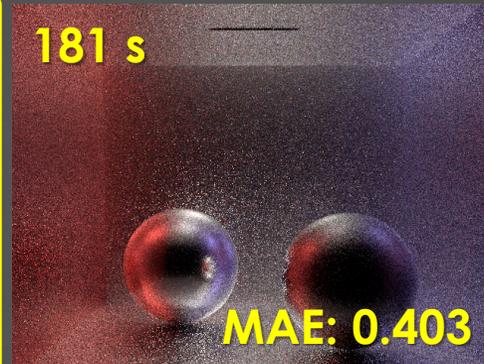
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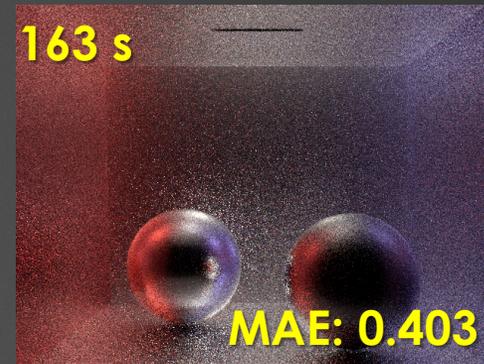
Precomputed



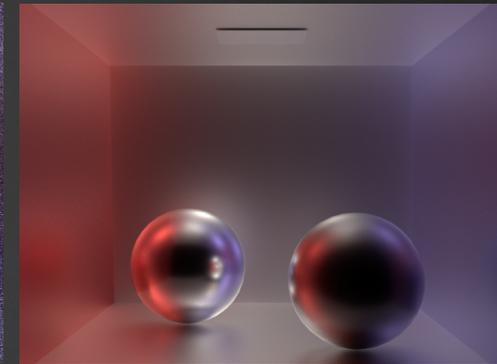
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Non-stratified  
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Semi-stratified  
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Reference

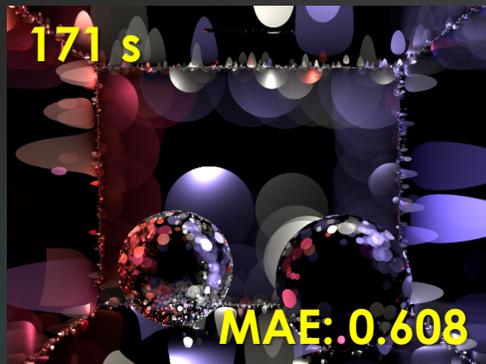
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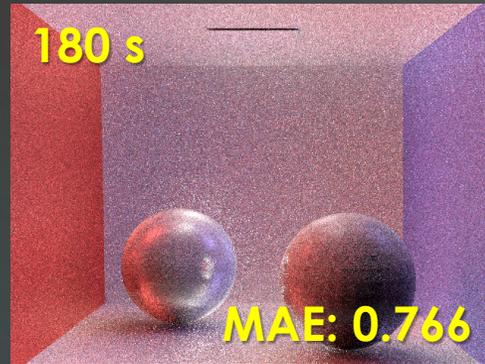
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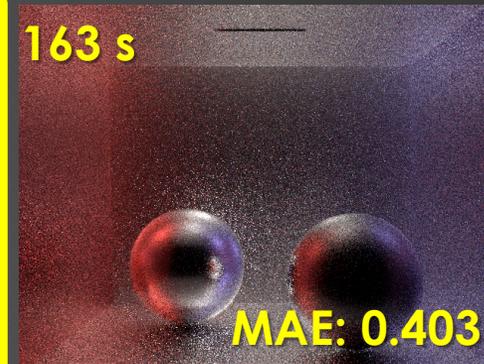
Precomputed



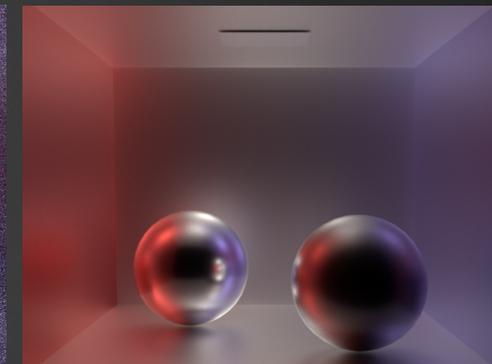
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(single precision)



Non-stratified  
(double precision)



Semi-stratified  
(single precision)



Reference

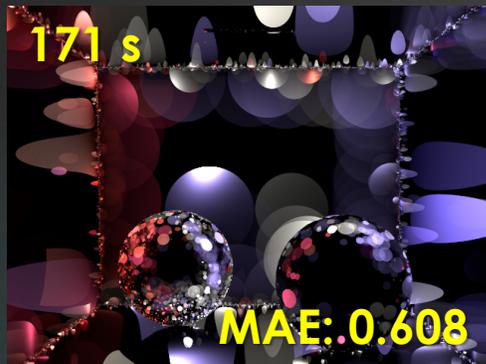
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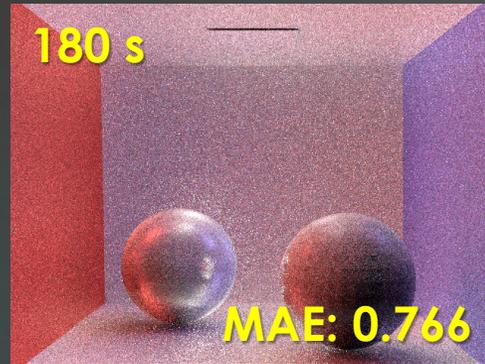
$$c_N^{-1}(\xi) = \frac{\xi}{N}$$

Fast & precise 😊

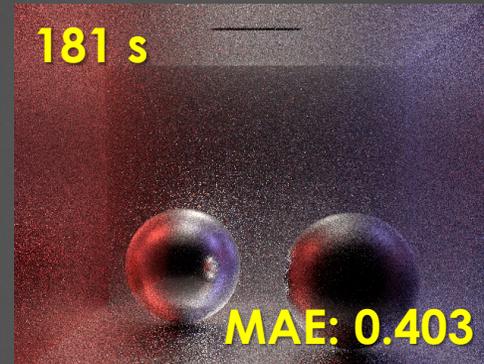
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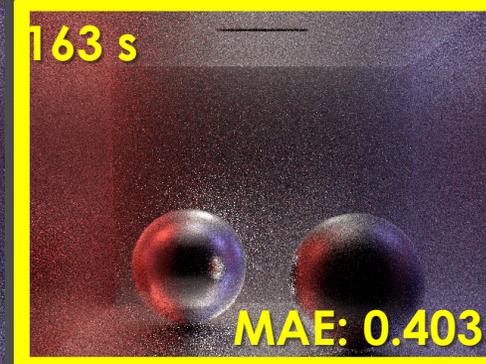
Precomputed



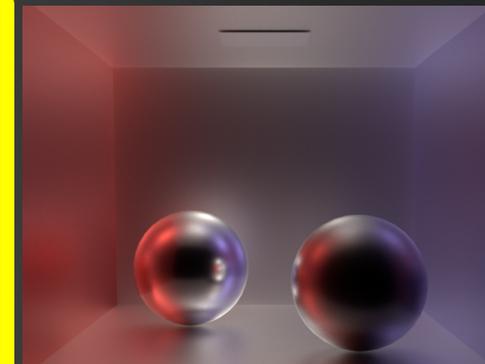
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Non-stratified  
(double precision)



Semi-stratified  
(single precision)



Reference

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Noticeable precision error for large  $N$  ☹️

$$c_N^{-1}(\xi) = \frac{\xi}{N}$$

Fast & precise 😊

# Combination with BPT



Regular vertex connection

+



Stochastic light culling  
(glossy reflection only)

=



Multiple importance sampling (balance heuristic) [Veach95]

# Equal-time Comparison (15 min)

AMD Ryzen™ Threadripper™ 1950X Processor



Caustics reflected on the mirror (GGX roughness: 0.0001)

# Limitations



- ▶ Perfectly specular surfaces
- ▶ Ellipsoidal range to bound refractions & anisotropic BRDFs
  - ▶ Future work: efficient approximations [Belcour18, Conty18, Xu13]
- ▶ Correlation of paths due to path reuse
  - ▶ Balance heuristic does not take this correlation into account
  - ▶ Future work: correlation-aware MIS heuristics [Popov15, Jendersie18]

# Conclusions

- ▶ Unbiased light vertex culling for BPT
  - ▶ Random range from each eye vertex
  - ▶ BVH of light vertices
- ▶ Decorrelation of variance
  - ▶ On-the-fly minimum random number generation in BVH traversal
- ▶ Limited to glossy reflections, but efficient for **very highly glossy** reflections
  - ▶ E.g., GGX roughness: 0.0001
  - ▶ Effective for specular-diffuse-glossy paths



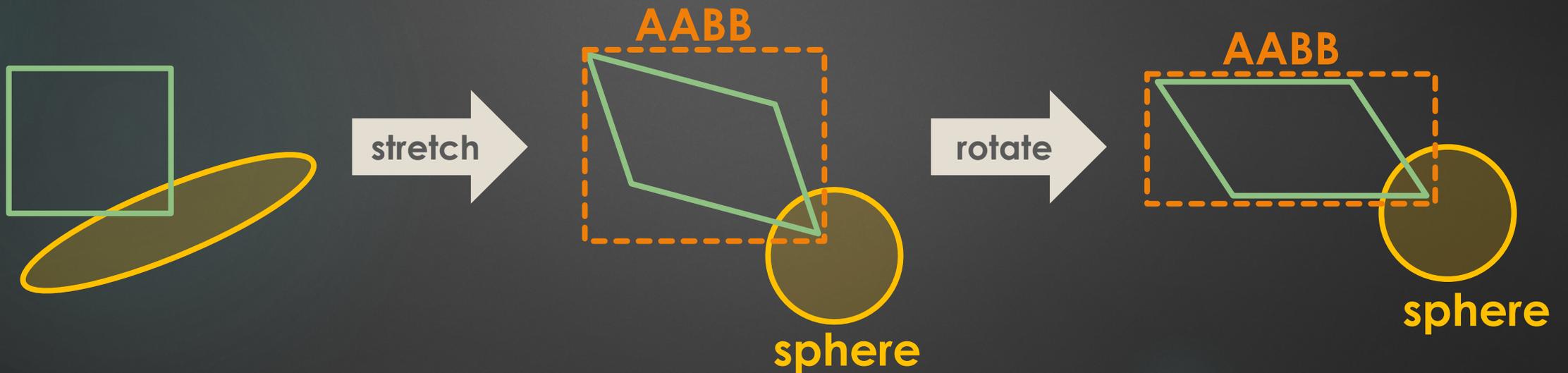
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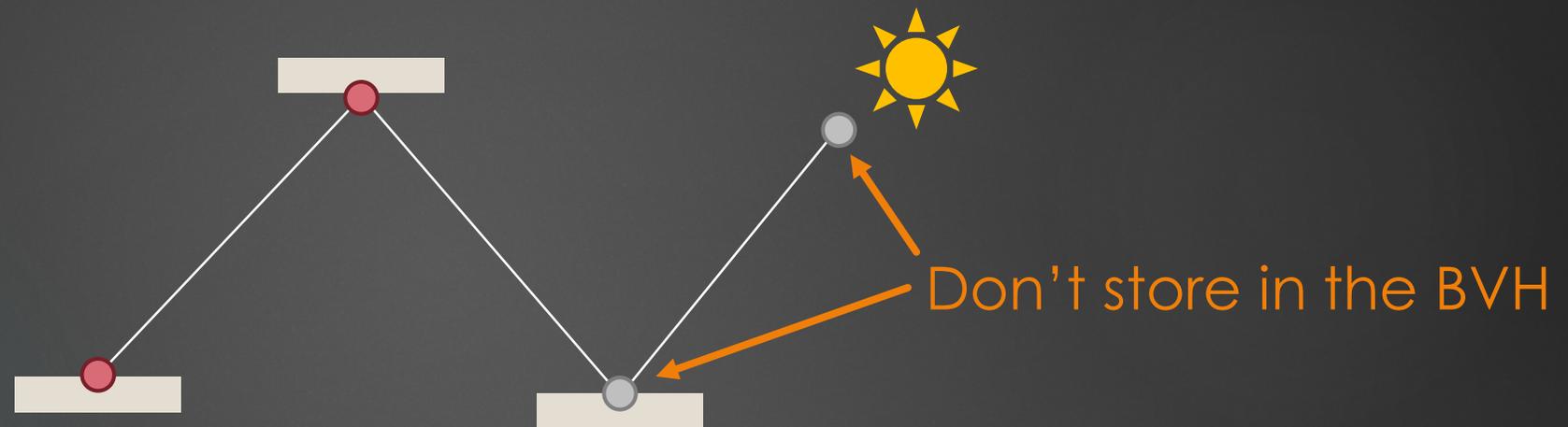
Backup

# Rough Intersection Test for Culling

- ▶ Exact ellipsoid-box intersection test is expensive
- ▶ Replace with sphere-AABB intersection test [Arvo90] in a stretched space
- ▶ Rotate the test space to minimize the AABB



# 1<sup>st</sup> and 2<sup>nd</sup> Light Vertices



- ▶ Exclude 1<sup>st</sup> and 2<sup>nd</sup> light vertices from the BVH
- ▶ Efficiently sampled by other techniques in BPT
  - ▶ E.g., next event estimation